

The fifth dimension as an analogue computer for strong interactions at the LHC

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ABSTRACT: We present a mechanism to get $S \simeq 0$ or even negative, without bringing into play the SM fermion sector. This mechanism can be applied to a wide range of 5D models, including composite Higgs and Higgsless models. As a realization of the mechanism we introduce a simple model, although the effect on S does not rely on the underlying dynamics generating the background. Models that include this mechanism enjoy the following features: weakly-coupled light resonances (as light as 600 GeV) and degenerate or inverted resonance spectrum.

KEYWORDS: Field Theories in Higher Dimensions; Beyond Standard Model; Technicolor and Composite Models.

Contents

1. The analogue computer in four steps	2
2. What holography has to say about the S parameter	3
2.1 A short account of the problem	3
2.2 Our solution	4
3. Some useful definitions	6
3.1 Relevant parameters	7
4. The effect of background fields on S	8
4.1 Holographic QCD: modifying the axial	8
4.2 A simple toy model: modifying the vector	11
4.3 A realistic example	11
5. Parameter scan for S	13
6. Phenomenology	14
7. 4D interpretation	18
7.1 UV independence and IR robustness	18
7.2 Purely 4D argument	19
8. Conclusions	21
A. From background fields to effective metric	22
A.1 Neutral scalar	23
A.2 Symmetry-breaking by a bulk scalar	24
B. Derivation of S in any holographic model	26
C. NDA for the condensates	28
D. Comparison 4D resonance saturation models	29
E. Link with TC scales	30

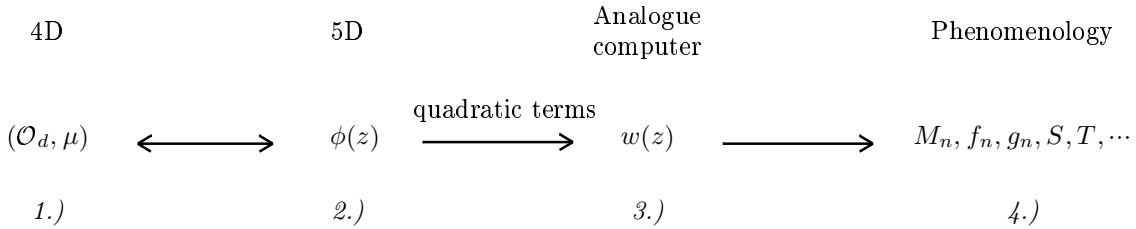


Figure 1: The role of the the analogue computer.

1. The analogue computer in four steps

Before we give a more formal Introduction in section 2, we briefly describe the purpose of this paper, as summarized by the four steps of figure 1.

1. Assume that a strongly-coupled sector is (partly or completely) responsible of electroweak symmetry breaking (EWSB). The strong sector can be described by, for example, physical resonances (mesons) and, in general, by a set of operators with couplings to the EW sector: $\{\mathcal{O}_d\}$. d denotes the scaling dimension of the operator and $\langle\mathcal{O}_d\rangle$ its vev.
2. Since computations in the strong sector are difficult, unreliable or impossible, one can use a tool, the fifth dimension, to extract physical quantities. The procedure is the so-called holographic recipe:¹ the properties of the set $\{\mathcal{O}_d\}$ are substituted by a set of bulk fields $\{\phi(z)\}$. This is achieved by relating the mass of the field to the dimension of the operator by $m_\phi^2 = d(d - 4)$ in units of the curvature scale, the running scale μ to $1/z$, the vev of $\langle\mathcal{O}_d\rangle$ to that of $\langle\phi\rangle$, and transformation properties of \mathcal{O}_d to the ones of ϕ ...

The degree of reliability of this procedure depends on the 5D gauge coupling, or in other words, on the corresponding 4D large- N expansion.

At this level, extracting physical quantities depends on one's ability to choose the right set $\{\phi\}$ and to solve coupled equations of motion. As you can imagine, this task is also rather lengthy, specially when several bulk fields are relevant for the discussion.

3. We propose to go a step further by realizing that, at the quadratic level, the effect of background fields on the resonances and Goldstone bosons is equivalent to introducing an *effective metric* and modifying the boundary conditions (BCs). Even when the background fields produce light modes [6, 7], one can still perform this rewriting while keeping the light fields in the spectrum. This is our analogue computer: whatever the background fields $\{\phi\}$, they result in a particular form of the effective metric felt by the mesons.

¹See [1–5] and references therein for more details on the correspondence conjecture.

This procedure, valid only at the quadratic level, is more relevant than it seems: except in particular cases [8], quadratic interactions are the only reliable quantities one can compute in these kind of models.²

4. Once the analogue computer is built, the rest is much easier: the scale of EWSB, the spectrum, decay constants, the couplings to the SM fermions and, in particular, the electroweak precision parameters like S and T can be computed straight away just in terms of a metric with few coefficients.

The effective metric has few coefficients because the effect of these background fields on observables decreases with the dimension d . In particular, only two condensates are relevant for the discussion, and S can be correlated with the properties of the strong sector mesons.

The paper is organized as follows. In section 2, we discuss the situation of S -parameter studies both from the 4D and 5D side. In section 3, we introduce the relevant parameters in our effective model. Section 4 goes into some details of the effect of background fields on S , and how the same physics is described by our analogue computer. Section 5 gives the result for S in the parameter space we consider. Section 6 describes the consequences on the spectrum. Section 7 presents a purely 4D interpretation of the result. We present our conclusions in section 8. Various appendices give details on technical points.

2. What holography has to say about the S parameter

Electroweak precision tests (EWPTs) seem to tell us that new physics models have to follow very determined patterns, as is the case for the SM itself. For example, one can suppress contributions to $\Delta\rho$ by enforcing custodial symmetry in the new sector [9]. This symmetry can be embedded into a larger symmetry that protects new couplings, like anomalous contributions to the $Z \rightarrow b\bar{b}$ [10].

2.1 A short account of the problem

We focus on the S parameter here, even though technicolor [11, 12] is also known to encounter other difficulties once it is extended to include fermion masses [13, 14]. In general, the new physics contributions to S are of order

$$S_{\text{tree}} \sim \frac{N}{4\pi} \left(\frac{v}{f}\right)^2, \tag{2.1}$$

where N is a measure of the size of the new sector [15]: it is an effective number of degrees of freedom. In (2.1), f/v represents the little hierarchy between the decay constant f of the three Goldstone bosons (GBs) that are eaten by the W 's and Z and the vev v of the composite Higgs (built up of pseudo-GBs of a larger group). In all known models, $(f/v)^2$ cannot be made much larger than a factor twenty [16, 17]. At the other extreme is the minimal case without a light Higgs. This is also the most disfavorable one: resonance and

²We'd like to thank Ami Katz for illuminating discussions on this point.

symmetry-breaking (SB) scales are then tied together, so that the formula (2.1) can be used with replacement $v \rightarrow f$.

As for N , it can be interpreted more specifically 1.) for a 4D strongly-interacting theory, as the number of colors or 2.) for a 5D model, as the number of Kaluza-Klein (KK) states that can be generated at low energies before strong coupling sets in. The relation between these two quantities is discussed in section 3.1.

The point is that, in either case, a large N is what makes the description in terms of resonances/KKs perturbative. Thus, in order for such scenarios to remain predictive at the few TeV scale, we need either a physical Higgs boson or a large N to ensure perturbativity. The first possibility can result in a small S : if there is a (composite) Higgs, the first resonance may be heavy enough as not to produce a large S , see [18] for the latest update. On the other hand, if the job of keeping WW scattering perturbative is to be done by the resonances alone, N cannot be too small, and one expects the lightest resonances below 2 TeV. We focus on this less favorable case here: the mechanism may then be applied to the more favorable situation with a composite Higgs.

The basic problem is that strong interactions readily produce large deviations from the SM. Among all the relevant electroweak parameters [19, 20], the main culprit is S . Whereas we know experimentally that $-0.4 \lesssim S \lesssim 0.2$ at the 3σ level [21], this bound is easily exceeded in scenarios of dynamical EWSB. Beyond using equation (2.1), possible methods of estimating S for strong dynamics include using QCD as an analogue computer [22, 15], combined with our best handle on strong interactions, namely the large- N limit. This yields $S \sim 0.1N$, so that a version of technicolor built as a simple rescaled QCD is experimentally excluded. Since only low- N theories would stand a chance of passing the constraint, computing S would be hopeless.

Such a picture was corroborated using a 5D approach in [23]. The 5D models in question are constructed to embody the same physics (confinement, symmetry-breaking) in a dual description in terms of mesons. For them to remain perturbative above the first few KK resonance (to be understood as the techni-mesons), the 5D gauge coupling should remain small in units of the AdS curvature. Since this quantity directly corresponds to the $1/N$ of a 4D theory, the 5D description fails for the same reason as the 4D one [24]. This is generically valid [25, 26, 7], unless the profile of the SM fermions is chosen to be nearly flat [27].

2.2 Our solution

In the present paper, we draw upon the 5D approach, without invoking cancellations with the fermion sector. In that approach, it is the bulk dynamics that generate large contributions to the S parameter (proportional to N). However, since there are two competing contributions with different signs — respectively from the vector/axial resonances — there is no generic value for S in a strongly-interacting model. In fact, we find that there is a significant fraction of parameter space for which S passes the experimental constraint.

The added ingredient compared to previous 5D modeling comes from holographic QCD [28, 6, 7]. Namely, we refine the 5D model by matching with the first terms in an OPE of the two-point functions [29]. In [30], we considered matching the 5D model to a

different high-energy behavior than that of QCD: we called this Holographic Technicolor. Here, we go further: we present a general parametrization of terms quadratic in spin-1 resonances. This allows us to correlate the experimental value of S with properties of the new physics sector. The result thus does not depend on the details of the underlying 5D modeling. Still, we provide as an example a model consistent with gravity, which implies definite signs and magnitudes for the parameters of our analogue computer. These allow for $S \lesssim 0$.

This parametrization serves as an analogue computer for strong interactions, and can be applied to 5D models with a physical Higgs scalar, such as composite Higgs models [17] or gaugephobic Higgs [31]. Note that, once such a model passes the constraint on S , the remaining experimental constraints on resonance masses come mainly from direct production. For numerical applications, we consider the extreme case where the lightest resonance has a mass of 600 GeV. It will turn out that a vanishing or slightly negative S is correlated with a degenerate spectrum, or even with an inverted spectrum, so the lightest resonance is a techni- a_1 rather than a techni-rho.

This study leads us to make the following claims. 1.) As was the case for 4D strong dynamics, the value of S cannot be predicted in general for 5D models. 2.) S can change sign in a weakly coupled 5D model [30]. 3.) Setting the value of S to be within the experimental bounds, one finds correlations between the spectrum, the couplings, the OPE and the scale of electroweak symmetry breaking.

Point 1.) is a known fact: although the natural estimate for S with light resonances is positive and order one, one can always rescue the particular model by switching on some compensating effects. This job becomes harder as the resonances are more weakly coupled (large- N), but was shown to be feasible in [30] and is further discussed here. Point 2.) is new in the sense that we are dealing with a weakly-coupled and light sector of resonances coupled to EWSB, and still we can reduce the value of S to be within experimental limits and even change its sign.³ Point 3.) is the subject of this paper: the use of the 5th dimension as a tool is very powerful to describe these correlations in a calculable way. In fact, we find that, in the simplest toy model for bulk fields one could write down, the cumulative effect of bulk dynamics can indeed go in the direction of lowering S , and go so far as to make it negative for reasonable values of the parameters.

As mentioned in section 2, the present paper discusses a class of model in which S_{tree} may vanish or even become negative. The second possibility is even more welcome for the following reason. The value of S is obtained by taking the difference between a model and the SM, used at loop level. The SM Higgs reference mass thus enters the calculation. The tree level contribution of (2.1) has to be corrected by the running loop effects [22, 15]. One can estimate these effects by running until the scale of new physics Λ

$$S = S_{\text{tree}} + \frac{1}{12\pi} \left(\ln \left(\frac{\Lambda^2}{m_H^2} \right) - \frac{1}{6} \right). \quad (2.2)$$

This effect is sizable and of order 0.1 for $\Lambda \sim 1$ TeV, so that we will require $-0.5 < S_{\text{tree}} <$

³We stress again that this occurs without resorting to cancellations with the fermion sector [32].

0.1. Therefore, rather than considering the bound $-0.4 \lesssim S \lesssim 0.2$ on S , we use a bound $-0.5 \lesssim S_{\text{tree}} \lesssim 0.1$ and omit the subscript “tree”.

3. Some useful definitions

Our analogue computer is a 5D model, where the KK modes can also be interpreted as the resonances of a strongly-interacting 4D theory. On the 5D side, we assume a conformally flat metric

$$ds^2 = w(z)^2 (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2), \tag{3.1}$$

where z is the extra coordinate, defined on an interval $l_0 \leq z \leq l_1$. Appropriate BCs will be enforced at the endpoints l_0 (the UV brane) and l_1 (the IR brane). $w(z)$ is the warp factor: $w(z) = 1, l_0/z$ corresponds to flat space and AdS respectively.

For applications to Holographic Technicolor, the interesting metrics are the so-called gap-metrics [30], which decrease away from the UV as AdS or faster. This warping of the metric is ultimately responsible of the existence of two sectors in the spectrum: the ultra-light (UL) sector consisting of the SM fields W, Z, γ and the Kaluza-Klein-sector (KK). The gap between them will be denoted in general as G .

We will use metrics that are asymptotically AdS on the UV boundary, and break conformal invariance near the IR

$$w(z) = \frac{l_0}{z} f\left(\frac{z}{l_1}\right), \tag{3.2}$$

where $f(0) = 1$.⁴ In most of the paper we will consider a simple parametrization of deviations from AdS

$$f\left(\frac{z}{l_1}\right) = \exp\left(\frac{o_{V,A}}{2d(d-1)} \left(\frac{z}{l_1}\right)^{2d}\right). \tag{3.3}$$

This can be obtained effectively by adding a LR kinetic term in the bulk with an appropriate profile, as in [30]. Section 4 and appendix A explain how two different effective metrics can be generated in a 5D model. Note that the phenomenology is not very sensitive to the particular form of $f(z)$ in the IR.

In the present paper, fermions are localized on the UV brane for simplicity. Therefore, the S parameter we compute here is a pure gauge contribution. In this way, flavor issues can be addressed separately from constraints on the S .

Now let us consider a $\mathcal{G} \supset \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L}$ bulk gauge symmetry. The LR symmetry is necessary for custodial symmetry [27]. It is also included in the $O(3)$ that suppresses deviations from the SM in $Z \rightarrow b\bar{b}$ [10]. The action is invariant under “parity” $L \leftrightarrow R$. We will denote the common $\text{SU}(2)_L \times \text{SU}(2)_R$ gauge coupling by g_5^2 , which has dimensions of length. The ratio between this and the $\text{U}(1)_{B-L}$ coupling \tilde{g}_5 can then be

⁴In general, we ultimately use $l_0 \ll l_1$ for numerical applications. Therefore, for all practical purposes, it does not matter whether one imposes $f(0) = 1$ or $f(l_0/l_1) = 1$.

chosen in order to reproduce the experimental M_Z/M_W . We will not need \tilde{g}_5 further in the present paper.

The breaking patterns that are relevant for phenomenology can be summarized as follows:

- $SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$ near/on the IR.
- $\mathcal{G} \rightarrow \emptyset$ or $U(1)_Y \times SU(2)_L$ on the UV brane for — respectively — Holographic QCD or the EW case.

3.1 Relevant parameters

Given the setup of the previous section, the size of deviations from SM Physics can be estimated by knowing:

1. The gap between the UL and KK sectors: $G \propto \left(\frac{M_{\text{KK}}}{M_W}\right)^2$
2. The size of the KK sector contributing to the EWSB sector: $N_{\text{KK}} \propto \left(\frac{4\pi f}{M_{\text{KK}}}\right)^2$

We discuss these in turn.

1. In a 5D model, G depends on the warping of space-time. If the only source of EWSB is via boundary conditions on the IR brane (Higgsless models) the value for G is simply given by a geometrical factor: G is just a number in flat space whereas it is a parametrically large factor — $\log(l_1/l_0)$ — for pure AdS. Large localized kinetic terms can increase G . For example, one can modify the spectrum in flat space by adding large localized kinetic terms in the IR brane, effectively mimicking a warp factor.
2. Using NDA in 5D, one can show [33–37] that the loop expansion for a 5D gauge field theory breaks down around the scale

$$\Lambda_{\text{UV}} = \frac{24\pi^3}{g_5^2}. \tag{3.4}$$

Using the standard definition for N

$$\frac{l_0}{g_5^2} \equiv \frac{N}{12\pi^2}, \tag{3.5}$$

which matches 4D and 5D correlators in the large energy limit, we can write

$$\Lambda_{\text{UV}} l_0 = 2\pi N. \tag{3.6}$$

In this language, large- N (4D) expansion corresponds to weak coupling (5D).

However, beyond the AdS case, the 5D expansion parameter N given by (3.5) does not coincide with the size of the low energy sector, N_{KK} . One has to realize that the result (3.4) holds for processes that would be localized on the UV brane, where the warp

factor is normalized to one. For metrics as in eq. (3.2), experiments carried out on the UV brane (where fermions are located) have a typical cutoff $2\pi N/l_0 \gg 2\pi N/l_1$. The result (3.4) gets redshifted for processes localized at a position z_* [5, 38, 39], i.e. if the involved overlap integrals are dominated by contributions around z_* . The scale at which a process localized in z_* becomes strongly-coupled is thus

$$\Lambda(z_*) = \frac{24\pi^3}{g_5^2} w(z_*). \tag{3.7}$$

For metrics of the form (3.2), this is

$$\Lambda(z_*) = \frac{N}{z_*} f\left(\frac{z_*}{l_1}\right). \tag{3.8}$$

This allows us to discuss the perturbativity of the model. If there were no particles except the UL modes, the scattering of these UL modes would become non-perturbative at energies of order $4\pi f$. However, light enough resonances can tame the amplitudes for scattering of light modes (as a Higgs boson would), yielding a model that remains perturbative until a higher scale Λ_{IR} . It turns out that a resonance spectrum starting at $4\pi f/\sqrt{N_{\text{KK}}}$ buys predictive power up to a scale given by $\Lambda_{\text{IR}} \sim 4\pi\sqrt{N_{\text{KK}}}f$ [40, 41]. This effective number of KK modes contributing to a given process is

$$N_{\text{KK}}(z_*) = Nf\left(\frac{z_*}{l_1}\right). \tag{3.9}$$

In AdS, because of conformal invariance, this turns out to be constant, and $N_{\text{KK}} = N$. For other warp factors, the effective size of the strong sector will depend on the energy scale as (3.9). The Λ_{IR} for scattering of light modes corresponds to using (3.8)–(3.9) with z_* of order — but usually smaller than — l_1 .

The generic picture is then that of figure 2, where the different scales are depicted. We have included some numerical values corresponding to the extreme case of section 6 with the lightest resonance at 600 GeV. Besides the massless photon, the spectrum consists UL modes, to be identified with the W^\pm and Z modes. The KK resonances starts at a higher scale (of order a few $1/l_1$), which is parametrically larger than M_W by a factor \sqrt{G} . There are N_{KK} resonances below the IR cut-off Λ_{IR} . The resonance spectrum is (approximately) equally spaced. The whole KK picture would break down at a scale Λ_{UV} , which is essentially Λ_{IR} times a blue-shift factor of order $(Nl_1)/(N_{\text{KK}}l_0)$.

4. The effect of background fields on S

Here we would like to illustrate claims 1.) and 2.) of section 2.2, namely that there is no prediction for S in 5D models and that S can change sign. In section 6, we will show that one can turn the experimental value of S into predictions for the new physics sector.

4.1 Holographic QCD: modifying the axial

Let us jump to GeV physics: we start by the case of Holographic QCD considered by [6, 42]. A bulk field (representing the quark condensate) triggers chiral symmetry breaking

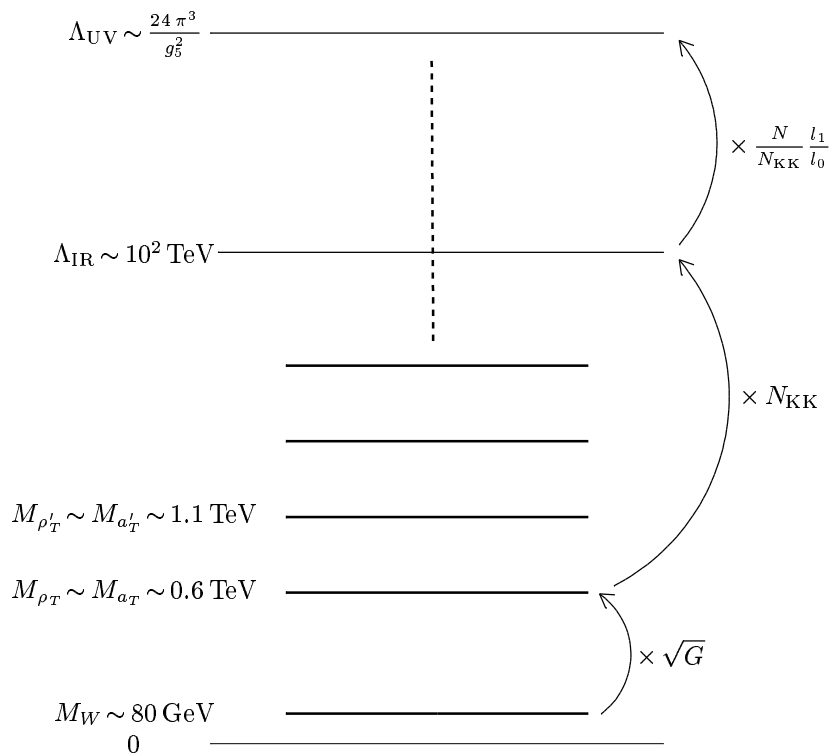


Figure 2: Schematic representation of the spectrum and relevant quantities (not to scale).

by coupling to the axial sector, and modifying its profile. Since this distinguishes the vector from the axial fields, chiral symmetry is broken. See appendix A.2 for details.

What is the effect of an order one change of the condensate (background field vev) on S ? The exact derivation is presented in appendix B, but the effect is an order one change in the value of S . In figure 3 we use the model of [42] which we feed into our analogue computer using appendices A.2 and B. We plot the value of S as a function of $o_A = \frac{15\pi^3}{N} \alpha_s \langle \bar{q}q \rangle^2 l_1^6$. The specific value $o_A \simeq 16$ used by [42] in a fit to QCD data is represented by a star. Note that the aim of Holographic QCD was not to predict the value of $S = -16\pi L_{10}$, but to extract it from data and correlate it with other observables.

One thing to notice is that, although one cannot predict the particular value of S , adding the effect of chiral symmetry in the picture always leads to a positive value of S . Here we see again how models with purely rescaled QCD are not able to pass the EWPT unless N is really small — such models would not be computable in the 5D picture [23].

The second question one has to address is: how sensitive is the value of S to the particular modeling of the IR physics? In figure 4 we compare the value of S computed in a model with a dynamical scalar coupled to the axial (as in [6, 42]) with the same S computed by simply adding an exponential profile to the metric itself (eq. (3.3) with $o_V = 0$). In both cases we have used Dirichlet BC for the axial fields on the IR brane. S changes by a few percent. This is also true for the spectrum: changing the parametrization (3.3) does not affect phenomenology much.

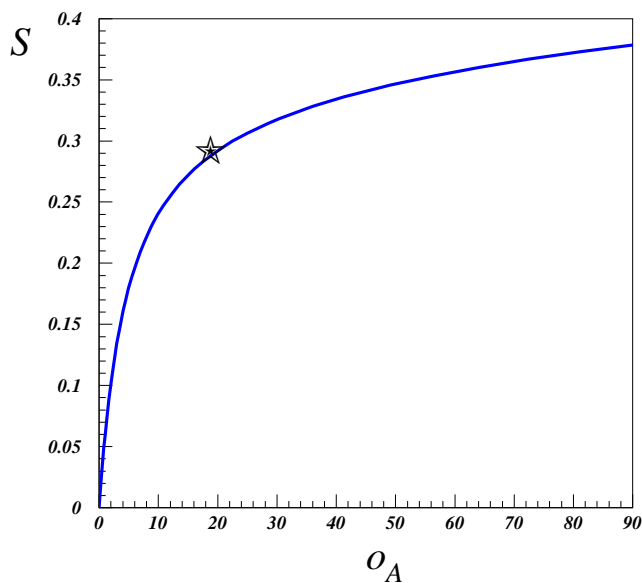


Figure 3: $-16\pi L_{10}$ for the Holographic QCD model of [7], as a function of o_A (ξ^2 in their notation). The best fit to QCD in [7] is $o_A \simeq 16$, as depicted by the star.

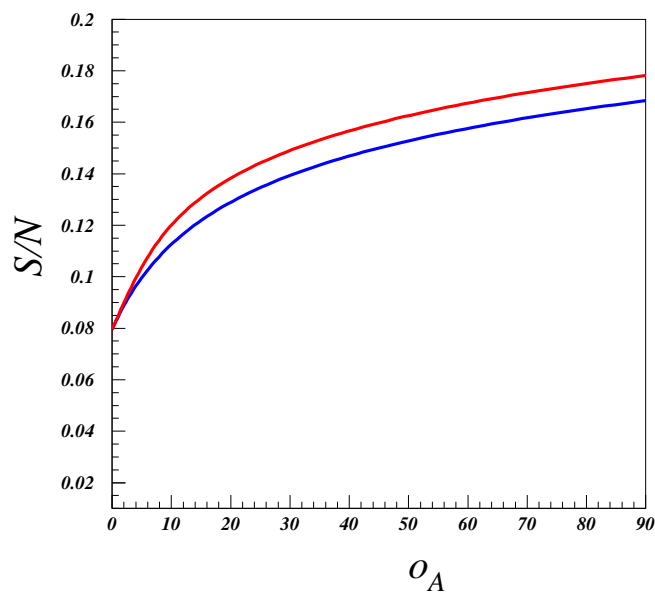


Figure 4: S/N vs o_A for $2d = 4$. The figure shows the exponential Ansatz (upper curve) and the exact hypergeometric result (lower curve). Both cases are computed assuming the large-condensate approximation (Dirichlet IR BC).

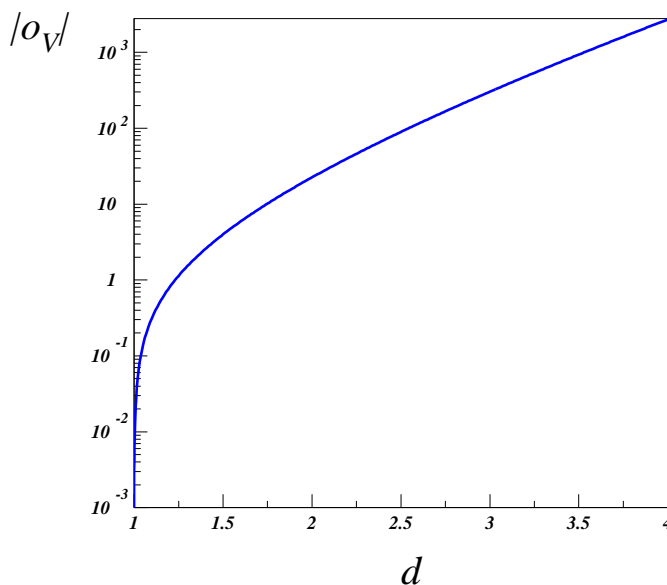


Figure 5: Magnitude of o_V necessary to invert the sign of S , as a function of the dimension d , with $o_A = 0$.

4.2 A simple toy model: modifying the vector

Now let us consider a completely different case: imagine it is the vector, not the axial, who feels the effect of a condensate. We thus set $o_A = 0$ in eq. (3.3). The value for S can be computed analytically. For negative o_V we get (see appendix B and [30])

$$S(o_V) = \frac{N}{4\pi} \left(1 - \frac{2}{3d} (\Gamma(0, \nu) + \log(\nu) + \gamma_E) \right), \tag{4.1}$$

where $\nu = -o_V / (2d(d-1))$.

Note that S can be either positive, or negative. We can now ask how easy it is to fix S to be very small. The answer depends on the dimension of the condensate (mass of the 5D field). In figure 5 we represent the necessary value of $|o_V|$ to yield $S = 0$ from eq. (4.1). The higher the dimension d , the more difficult is for the field to produce an effect on observables. The natural size for o can be judged from the Holographic QCD result $o_A \simeq 16$ [7]. In other words, a high dimension condensate is too peaked towards the IR brane in order to give a sizable effect. Thinking on localized terms on the IR as infinite dimension condensates already tells you that they cannot help lowering the S (their NDA size implies a small effect, unless N is small).

4.3 A realistic example

We saw in section 4.1, figure 4 that the value of S is quite insensitive to the modeling of deviations from conformality. Neither is the spectrum or the couplings to SM fields. Therefore, we can parametrize the breaking of AdS conformal invariance as in (3.3).

Note that the particular dynamics generating the values of o_V and o_A is irrelevant for phenomenology.⁵ We just want to show here an explicit example of a natural theory leading to these effects.

As an example of dynamics capable of producing such deviations from conformality, we consider the following action

$$\begin{aligned}
 S = & \frac{1}{2\kappa^2} \int d^5x \sqrt{g} \left(-\mathcal{R} - V_\phi(\phi) + \frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi \right) \\
 & - \frac{1}{4g_5^2} \int d^5x \sqrt{g} g^{MN} g^{RS} \langle L_{MR} L_{NS} + R_{MR} R_{NS} \rangle \\
 & + \frac{1}{2g_5^2} \int d^5x \sqrt{g} (g^{MN} \langle D_M X D_N X \rangle - V_X(X)), \tag{4.2}
 \end{aligned}$$

where $\langle \dots \rangle$ means the trace in flavor space, and $R_{MN} \equiv \partial_M R_N - \partial_N R_M - i[R_M, R_N]$. The square of the 5D YM coupling g_5^2 has dimensions of length. κ is the 5D Newton constant related to the curvature l_0 and the bulk cosmological constant Λ by $1/l_0^2 = -\kappa^2 \Lambda/6$.

Essentially, the action (4.2) contains the effect of two background fields: ϕ affects gravity $g(x^\mu, z)$ and X mostly affects the Yang-Mills field $A \propto L - R$. In appendix A, we derive solutions for this model and show that the net effect can be absorbed into effective metrics. In particular, we show that the effect of ϕ and X on $w_{A,V}$ is the following:⁶

1. Common background: the real scalar $\phi(z)$ coupled to gravity will produce an effect common to vector and axial ($w_A = w_V$). If ϕ is non-tachyonic,⁷ the effect goes in the direction

$$\phi \text{ non-tachyonic} \implies o_V^\phi = o_A^\phi < 0, \tag{4.3}$$

i.e., it shuts off the IR part of the geometry — see appendix A.

2. Symmetry-breaking by a bulk scalar: charged scalar X coupled to the axial sector. At the quadratic level, the breaking of the $SU(2)_L \times SU(2)_R \longrightarrow SU(2)_V$ by a bulk scalar is equivalent to introducing an effective metric for the axial channel, and modifying the BCs. The Goldstone bosons eaten by the W, Z is a combination of the A_5 and of the zero mode of the radial part of X . This effect predicts a definite sign

$$o_V^X = 0, \quad o_A^X > 0 \tag{4.4}$$

and results in $w_A > w_V$.

For example, if the background is AdS and if X has a constant 5D mass, $\langle X \rangle$ is a power-law

$$\langle X \rangle = \sigma z^d, \tag{4.5}$$

⁵For example, in [30] we simply added a LR term parametrizing $w_A - w_V$ at the quadratic level.

⁶We neglect the dynamics of the fields responsible of the modifications [43], except the light modes. See discussion after eq. (B.8).

⁷See [44] for a different approach.

and we get

$$w_X(z) = \frac{l_0}{z} F_1 \left(; \frac{d-1}{d}; \frac{\sigma^2 l_0^2 z^{2d}}{2d^2} \right)^2 \stackrel{d=2}{=} \frac{l_0}{z} \text{Cosh}^2 \left(\frac{\sigma z^2}{2} \right). \quad (4.6)$$

3. Adding several fields of scaling dimensions $2, 3, 4 \dots d$ would have an effect on the metric suppressed by $(z/l_1)^{2d}$. The lower the dimension, the more the deviation from AdS extends inside the bulk. The effect is of course maximum on the IR brane, but still there it is suppressed by $d(d-1)$ as the dimension of the condensate increases. See appendix C.

To conclude, 1.) for phenomenological purposes, one only needs to consider the effect of the lower dimension condensates, 2.) the vector channel is only affected by the neutral scalar ϕ , with a definite sign

$$o_V < 0, \quad (4.7)$$

whereas the axial channel is also affected by the charged scalar, and the two effects compete, resulting in

$$o_A > o_V. \quad (4.8)$$

This is also Witten's positivity condition obtained from the spectral functions of 4D theories [45].

5. Parameter scan for S

In this section we study the parameter space that leads to small S . The key point is that one can study this issue and correlate it with the spectrum. Thanks to our analogue computer, this can be done without going into the detailed dynamics that produced the deviations. Studying these correlations will be the point of section 6.

The importance of encoding the effects of various background fields into effective metrics is that it simplifies the task of computing (4D) observables. Many 4D quantities involve contributions from all KK modes. Using the effective metrics, such sums can be expressed as simple integrals over the fifth dimension. To summarize, a Sum Rule (SR) works as follows

$$\begin{aligned} \text{relevant 4D quantities} &= \sum_{\text{KKs}} \text{KK properties} \\ &= \text{geometrical factor.} \end{aligned}$$

The beauty of the SR is to relate the sum over KK contributions with a pure geometrical factor that can be computed with just the knowledge of the metric. This is an advantage because sampling KK properties over a whole parameter space would be a herculean task. Namely, if deviations of AdS are included, one would have to solve numerically the equations of motions for at least the low-lying states and extract the masses and couplings.⁸

⁸In addition, it will turn out that, for models yielding $S \simeq 0$, one needs to take many states into account before noticing that the vector and axial contributions to S cancel out.

The S parameter is a good example of a SR: we have

$$\begin{aligned}
 S &= 4\pi \sum_n f_{V_n}^2 - f_{A_n}^2 \\
 &= \frac{N}{3\pi} \int_{l_0}^{l_1} \frac{dz}{l_0} (w_V(z) - w_A(z)\alpha^2(z)),
 \end{aligned}
 \tag{5.1}$$

where $\alpha(z)$ is the wavefunction of the GBs and it is again purely geometrical

$$\alpha(z) = 1 - \frac{\int_{l_0}^z \frac{dz'}{w_A(z')}}{\int_{l_0}^{l_1} \frac{dz}{w_A(z)}},
 \tag{5.2}$$

and $0 < \alpha < 1$. If instead of breaking chiral symmetry by BCs, one uses the bulk scalar $X(z)$, $\alpha(z)$ is slightly modified (see appendix B, eq. (B.8)).

The result (5.1) can also be understood by using the original definition for S [15]

$$S = 2\pi \frac{d}{dQ^2} (Q^2 \Pi_V(Q^2) - Q^2 \Pi_A(Q^2)) \Big|_{Q^2=0},
 \tag{5.3}$$

which is the difference between the kinetic terms generated for the V and A sources. These two terms correspond to the two terms in (5.1) as can be understood from the following. Non-zero sources generate a field $\Phi_{V,A}(z)$ in the bulk, yielding a 4D kinetic term $\int dz/g_s^2 w_X(z) \Phi_X(z)^2$. Now, the Φ 's obey the standard IR BCs, but are subject to the UV normalization appropriate for sources $\Phi_{V,A}(z) = 1$. Solving for the massless wave equation, one finds $\Phi_{V,A}(z) = 1, \alpha(z)$ respectively, which leads to the previous result (5.1).

Note that S is insensitive to the UV cutoff l_0 . This is what one should expect for a low-energy quantity coming from the strong sector.⁹

In our explicit example, the ϕ field affects both axial and vector while the effect of X goes in the opposite direction leading to the conditions $o_V < 0$ and $o_A > o_V$ (4.7)–(4.8). We are fortunate that the region of $S \leq 0$ lies in that region, see figure 6. In that figure, we show the region in parameter space where S changes sign. In pure AdS, $S = N/4\pi$ [47]. The authors of [24] realized that increasing the (common) warping would not change the sign of S , as you can realize by looking at the expression of S , eq. (5.1), with $w_A = w_V$ and noting that $\alpha \leq 1$. These authors also noted that one can make S small by going to the lower-left part of the diagonal (line B) in figure 6, but that this would require a low N .

Another direction explored by the authors of [7] is to increase o_A : this is depicted in figure 6 by the arrow A pointing along the $x > 0$ axis. The model of [46] should also lie on that arrow, but it treats the UV differently.

6. Phenomenology

From the study of the (o_A, o_V) parameter space performed in section 5, we found a region corresponding to $S \simeq 0$. We now extract the characteristics of models in that region.

⁹See [46] for a different approach (by keeping $l_1/l_0 \sim 6$).

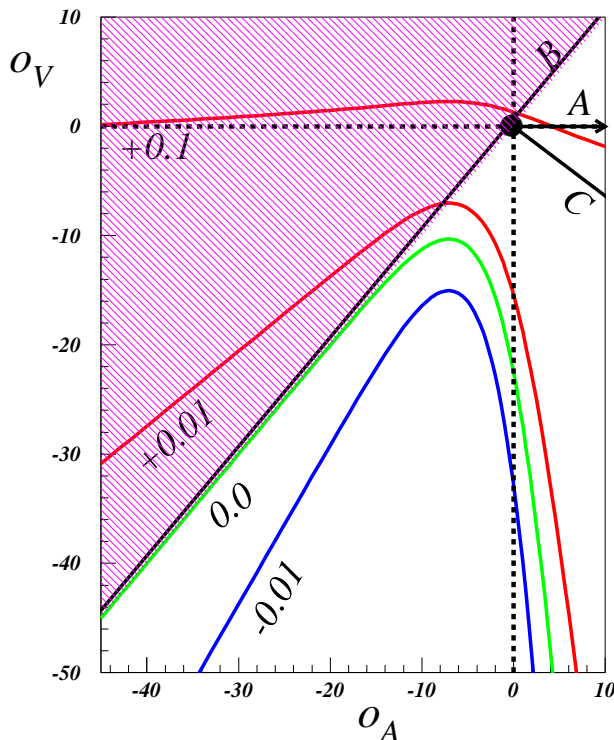


Figure 6: Contour lines for S/N in the (o_A, o_V) plane, for the case $2d = 4$. The dot at the origin represents the original warped Higgsless model [25]. Line B corresponds to having $w_A = w_V$, but with warping different from AdS, as in [24]. Line C denotes the class of models respecting the QCD factorization relation $o_V = -7/11o_A < 0$. The arrow A represents models with condensates only in the axial channel [7]. The shaded region is forbidden by Witten’s positivity condition.

To go further, we focus on the extreme case with the lightest possible resonances, $M_{a_T} = 600 \text{ GeV}$. Having such a light KK compared to M_W improves perturbativity (increases N_{KK}) as depicted in figure 2. We also have to match the Fermi constant G_F , or equivalently $f = 246 \text{ GeV}$. This allows us to fix enough parameters to draw the exclusion plot for $-0.5 < S_{\text{tree}} < 0.1$ in figure 7.

In the (narrow) left part of the band, one ends up in a situation where vector and axial resonances are nearly degenerate. In the right part of the band ($o_A > -10$), the spectrum is inverted respect to the QCD case: the axial resonance is lighter than the vector one. This is depicted in figure 8, (see where we plot the ratio M_ρ/M_{a_T} as a function of o_A , along the line of $S = 0$).

In figure 9 we depict the two situations we have just explained: degenerate or inverted spectrum for $o_A \lesssim -10$.

Note that, imposing $S = 0$ would imply an exact relation between o_V and o_A , $o_V = f(o_A)$. Passing the experimental constraint requires this relation to be fulfilled only ap-

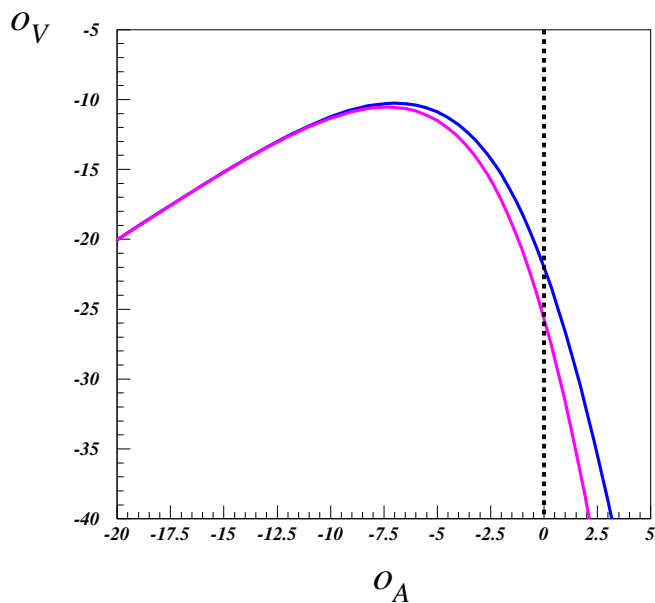


Figure 7: The allowed parameter space in the (o_A, o_V) plane in order to satisfy $-0.5 < S < 0.1$, after imposing $M_{a_T} = 600$ GeV.

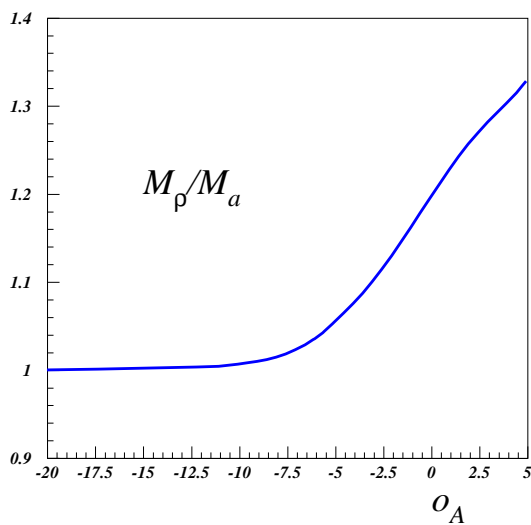


Figure 8: Plot of the ratio M_{ρ_T}/M_{a_T} along the line of $S = 0$, as a function of o_A .

proximately, as shown in figure 7. Obviously, an improvement in experimental constraints would select a narrower region of parameter space, but let us anyway quantify the adjustment. We see that, for the inverted case $o_A \gtrsim -10$, o_V only needs to be equal to $f(o_A)$ within 10%. The adjustment between the two potentials V_X and V_ϕ to obtain this relation

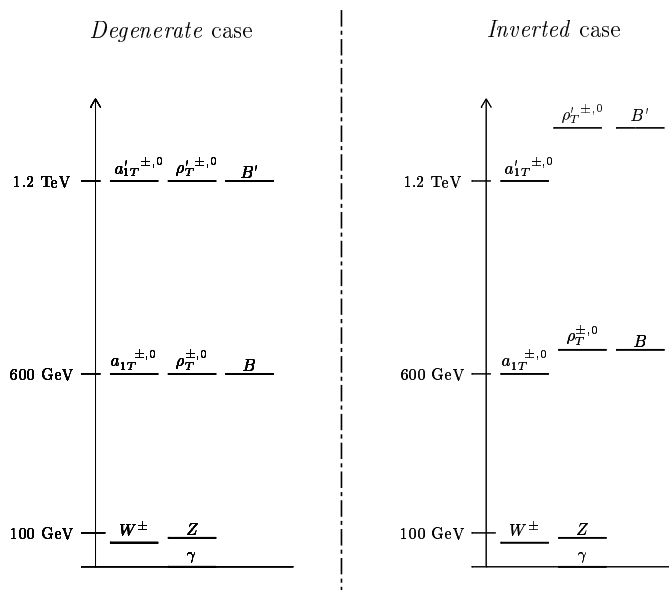


Figure 9: Schematic depiction of the spectrum (not to scale). The B resonances are the excitations of the $U(1)_{B-L}$ field.

Step	Requirement	Parameter to set	Predictions
0		Choose $o_A = 0$	
1	$M_{a_T} = 600 \text{ GeV}$	$l_1 \simeq 6.4 \text{ TeV}^{-1}$	$M_{A_{2,3,4}} \simeq 1.1, 1.6, 2.1 \text{ TeV}$
2	$S = 0$	$o_V \simeq -22.5$	$M_{V_{1,2,3,4}} \simeq 0.7, 1.35, 1.9, 2.4 \text{ TeV}$
3	$f = 246 \text{ GeV}$	$N = 146$	$f_{V_{1,2,3,4}} \simeq 13.8, 8.7, 1.9, 2.4$ $f_{A_{1,2,3,4}} \simeq 12.3, 9.0, 7.5, 6.5$
4	$M_W = 80.4 \text{ GeV}$	$\log(l_1/l_0) \simeq 4.5$	resonance isospin splittings, T

Table 1: Step-by-step flowchart for our particular benchmark model with $S = 0$.

is at the same level as the that of the radion potential [48, 49]. For $o_A \lesssim -10$, o_V would have to be equal to $f(o_A)$ within 1%. Remember however, that this is the case where vector and axial resonances are degenerate, which could come in 4D from a symmetry [50–52].

We have focused up to now on S , which involves contributions from all resonances, and could thus be expressed through a SR. The same is true for G_F . All of this is done automatically within the analogue computer. Since the effect of background fields can be encoded into a effective metric, at the quadratic level, we can also use this parametrization to extract the decay constants and the masses. Table 1 shows the successive steps that lead to predictions. In that case, we have to proceed again for each point in parameter space.

The 5D parameters necessary to describe the new physics sector are the coupling

constant $1/N$, the scale of KK resonances $1/l_1$, and the two condensates $o_{A,V}$

5D parameters : N, l_1, o_A and o_V .

The procedure we follow to fix them in terms of 4D parameters is to use the value of f and fix the lowest resonance to be at 600 GeV (bounds from TeVatron [53] and LEP [19]). Thus, the value of N and l_1 are just functions of o_A

$$f = 246 \text{ GeV}, M_{W'} \sim 600 \text{ GeV} \implies N(o_A), l_1(o_A).$$

We also set S within the experimental range, leading to the determination of o_V as a function of o_A ,

$$S \implies o_V(o_A).$$

Note that in order to obtain a good approximation for S by summing up resonances, one needs to take into account the resonances up to $\mathcal{O}(10)$ TeV.

This predicts any other observable in terms of one parameter, o_A . On the other hand, a natural potential roughly sets $|o_A| \lesssim 100$. To illustrate how this proceeds, we show in table 1 the various steps for a model with $S = 0$. To fix numbers, we need to pick one value for o_A . We choose $o_A = 0$ for simplicity. Note that, except for step 4, we do not need to specify l_0 : everything is finite in the limit $l_0 \rightarrow 0$, and would only receive small corrections. It is the value of M_W that sets l_0/l_1 .

7. 4D interpretation

7.1 UV independence and IR robustness

The S parameter can be written as the difference between a vector and an axial contribution, see (5.1). While both terms in (5.1) are dominated by the UV, the difference is finite as $l_0 \rightarrow 0$, as should be: the S parameter is insensitive to the UV details of the model, since the chiral symmetry is restored at high energies. This is embodied in the 5D model by

$$w_V(l_0) = w_A(l_0), \tag{7.1}$$

so that the UV does not contribute to the S parameter, as can be seen in figure 10.

In this figure, we show the value of $w_V - w_A \alpha^2$ as a function of the bulk coordinate z . Notice how, in the AdS case, the contributions come mostly from the IR region, whereas for the cases of interest, (smaller) contributions compete against each other, and come from the whole bulk. As a consequence, the low-energy quantity S is independent of the behavior of the two-point functions at very high energies. This also implies that we will not be sensitive to the high resonances, and to whether their spectrum follows the Regge behavior or not. In summary, the 5D model needs to match the assumed OPE of the two-point functions only for intermediate energies, not for asymptotically large ones.

Another key point is that the precise form of the deviations near the IR is not important. Indeed, the integral expression (5.1) for S receives most of its contributions from

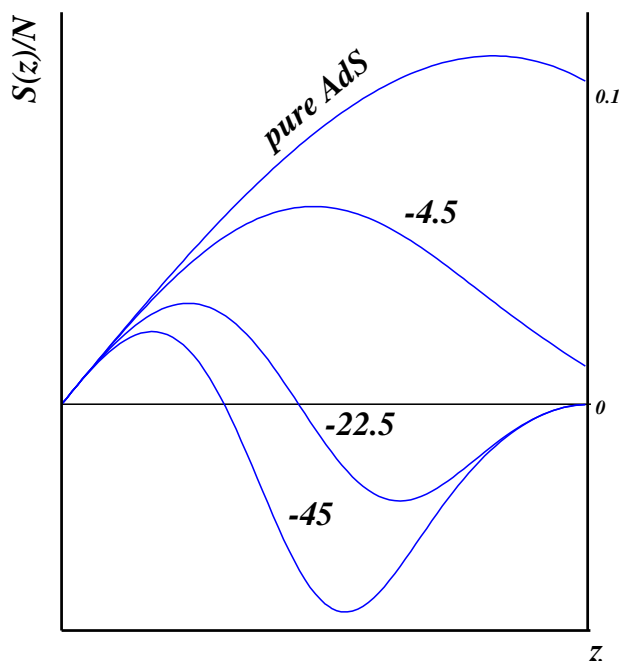


Figure 10: Value of the integrand yielding S in the sum rule (5.1). Indicated is the value of o_V , assuming $o_A = 0$ for simplicity.

the bulk. Therefore, a strong suppression of the metric near the IR is not important for the result. What is essential is that the condensate in the vector channel be large enough for $z \sim l_1/\text{few}$. We have indeed checked the robustness of our results when using different Ansätze for the metric instead of (3.3) (see for example figure 4).¹⁰

Finally, note the following from figure 10. In the AdS case, the contributions to S come from the IR. In that case, S can be estimated by including only the contribution from the lightest resonance. For $o_V \simeq -22.5$, which leads to $S \simeq 0$, all the intermediate energies contribute — and cancel out. In this sense, the result for S includes more contributions from intermediate energies than in the QCD case: one needs to sum up resonance contributions up to $\mathcal{O}(10)$ TeV to realize that S cancels out. This is somewhat expected from the 4D side [55, 56].

7.2 Purely 4D argument

In this section, we describe in which way the condensates and the S parameter are correlated, by simply considering the left-right two-point function. This explains the result of section 5, independently of the 5D modeling. This can be done with the help of figure 11,

¹⁰In this sense, the discussions of [44], putting the emphasis on the extrapolation of the OPE to the IR in order to generate confinement, and [54], describing the dependence of the spectrum of heavy resonances on the shape of the IR cut-off are not our concern when discussing the S parameter.

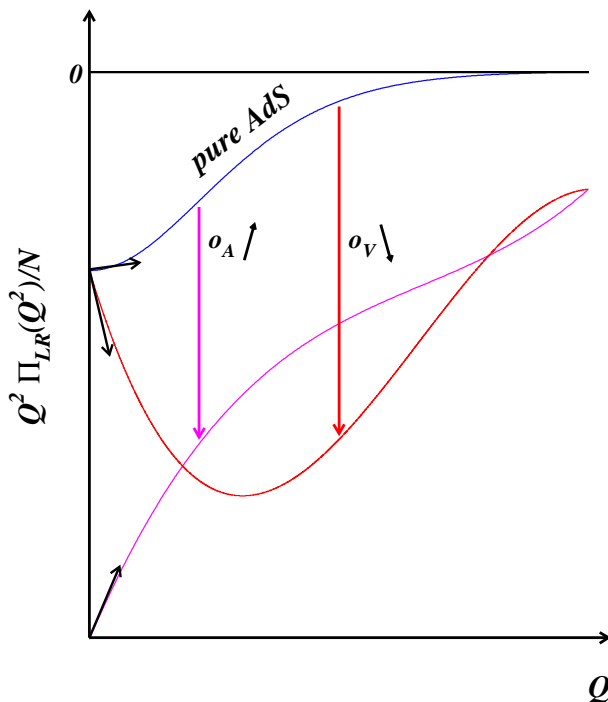


Figure 11: Cartoon of the left-right two-point function, showing the influence of axial and vector condensates on the slope at the origin, S .

where $Q^2 \Pi_{LR}(Q^2)$ is depicted: the asymptotic behavior of the curve is given by

$$Q^2 \Pi_{LR}(Q^2) \Big|_{Q^2 \rightarrow +\infty} = \frac{\langle \mathcal{O}_V - \mathcal{O}_A \rangle}{2Q^{2(d-1)}} < 0, \quad (7.2)$$

which has to be negative in order to fulfill Witten's positivity condition, which states that the whole function should be negative [45]. Also, the GB decay constant can be defined from the intercept at the origin

$$Q^2 \Pi_{LR}(Q^2) \Big|_{Q^2=0} = -f^2. \quad (7.3)$$

Like any 4D model, the present model provides an interpolant between these two regimes, using a set of spin-1 resonances, while satisfying basic field-theoretical requirements. Also, as a bonus compared to earlier models, it includes the perturbative behavior of both two-point functions Π_V and Π_A separately. This has an incidence, as the result does not depend only on the difference $o_V - o_A$, but on the two variables o_V and o_A .¹¹

Figure 6 shows that obtaining $S < 0$ requires $o_V < 0$. This can be understood in general, without resorting to the 5D model as follows. Start from the fact that the standard QCD case, or even the case $o_V = o_A = 0$ lead to $S > 0$. The upper curve in figure 11 represents $Q^2 \Pi_{LR}(Q^2)$ for this latter case. Making o_A positive would bring the curve down in the IR according to the asymptotic behavior (7.2). This is however not enough to make

¹¹To get an idea of the way things work in simple 4D models see appendix D.

the slope at the origin (S) negative. This is because the intercept $-f^2$ also goes down with larger o_A : f^2 is the decay constant of the would-be GBs, and is therefore sensitive to the condensates in the axial channel. The outcome is depicted by the second curve in figure 11, which also has a positive slope at the origin. On the other hand, decreasing o_V would also bring the curve down in the UV, but this time without modifying the intercept at the origin. In this case, one can get a negative slope at the origin, $S < 0$, third curve in figure 11. The same kind of reasoning would also show that the effect is greater with a low-dimension condensate.

8. Conclusions

We have presented an effective parametrization of quadratic interactions between spin-1 KK resonances, in a 5D model defined on an interval. The key point is that, at the quadratic level for the gauge fields, any coupling of a background field can be recast as an effective metric. This is even true in the case of background fields with light excitations [6, 7]. We have also displayed how the rewriting works in examples of explicit 5D models including background fields.

Though this rewriting may be performed for one's favorite model, we focus on the next step: we start from our generic parametrization, and consider the physical consequences independently of the details of the underlying dynamics. In this respect, the parametrization used here is an analogue computer: it allows us to study the correlations between observables. Here, we considered the interplay between the spectrum and the S parameter.¹² We have performed this analysis with the simplest modeling of the IR cut-off, and explored the parameter space that leads to a phenomenologically viable $S \simeq 0$. It turns out that the results do not depend on the deep IR modeling, but rather on the behavior for the whole range of intermediate scales. This is expected from discussions of walking in 4D technicolor [55, 57, 56]. The result for the low-energy parameter S is also UV-insensitive, as should be.

There is a common lore that the S parameter constraint excludes strong interactions as the source of EWSB. Our results strengthen the objections to this claim. What is indeed true is that strong interactions do generate large — proportional to N — contributions to S . Still, these contributions do not have a fixed value, but strongly vary in the parameter space we have explored. In fact, we find that cancellations between the vector and axial contributions to S do occur for reasonable values of these input parameters.

Let us address the question of fine-tuning. The analogue computer is a tool that parametrizes the effects of background fields on phenomenology without relying on particular dynamics. In this context, questions on fine-tuning of parameters are meaningless: whether there is a model that predicts some particular values for the metric is out of the scope of this approach. The point of a 5D model is not to predict the value of S , but to correlate the experimental value for S with properties of the strongly coupled sector observables. We have examined the region of parameter space corresponding to $S \simeq 0$, and shown that it corresponds to having the axial resonances either degenerate with (as already

¹²In a paper in preparation we explore other correlations between observables and the S .

considered in [51]), or lighter than the vector ones. We stress that the scenarios we have considered have a sufficiently large N to enjoy a weakly-coupled mesonic/5D description until a few tens of TeV. To achieve this, the lightest resonance (the techni- a_1) should appear below a TeV (maybe as low as 600 GeV), which is now allowed since the constraint on S is lifted. Also, we repeat that the mechanism of cancellation can be equally well applied to the present extreme Higgsless case as to composite or gaugephobic Higgs models.

Another point of our analogue computer is that its effective parameters are directly related to terms in the 4D OPE. This allows us to point to specific directions in the space of 4D strong interactions. Admittedly, devising a 4D mechanism that generates these effective parameters dynamically will be a much harder enterprise. Still the following statements can be made. To obtain $S \lesssim 0$, one needs a significant departure from AdS in the bulk — not just on the IR brane. In the 4D picture, this is tantamount to having a low-dimension condensate with a sizable magnitude. It may be that walking effectively produces such a low-dimension scaling, via the large anomalous dimension that the quark condensate acquires. To clarify a possible connection with walking, we have thus shown how to translate the various scales of the 5D model into those of extended and walking technicolor.

Still, to obtain $S \simeq 0$, the relative values of the condensates appearing in the OPE of the V & A correlators need to be altered with respect to the QCD case. How this happens in a technicolor model is unclear to us. On the other hand, we have shown that the respective values of the condensates $o_V < 0$ and $o_V < o_A$ would follow from the simplest 5D modeling. This is quite encouraging, and needs to be studied further, especially in connection with a possible dual 4D description.

In this paper we described a scenario with fermions located on the UV brane, although a natural setup would have bulk fermions, leading to a more interesting phenomenology.

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A. From background fields to effective metric

Consider the action involving gravity, a set of $SU(N_f)_L \times SU(N_f)_R$ Yang-Mills fields, a scalar X charged under the gauge symmetry as (N_{fL}, N_{fR}) and a neutral scalar ϕ ,

$$\begin{aligned}
 S = & \frac{1}{2\kappa^2} \int d^5x \sqrt{g} \left(-\mathcal{R} - V_\phi + \frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi \right) \\
 & - \frac{1}{4g_5^2} \int d^5x \sqrt{g} g^{MN} g^{RS} \langle L_{MR} L_{NS} + R_{MR} R_{NS} \rangle \\
 & + \frac{1}{2g_5^2} \int d^5x \sqrt{g} (g^{MN} \langle D_M X D_N X \rangle - V_X)
 \end{aligned}
 \tag{A.1}$$

where $\langle \dots \rangle$ means the trace in flavor space, and $R_{MN} \equiv \partial_M R_N - \partial_N R_M - i[R_M, R_N]$. The square of the 5D YM coupling g_5^2 has dimensions of length. The action (A.1) is invariant under “parity” $L \leftrightarrow R$ and the 5D $SU(N_f) \times SU(N_f)$ gauge transformations denoted by $R(x, z), L(x, z)$ acting as $R_M \equiv R_M^a T^a \mapsto R R_M R^\dagger + iR \partial_M R^\dagger$. κ is the 5D Newton constant related to the curvature l_0 and the bulk cosmological constant by $1/l_0^2 = -\kappa^2 \Lambda/6$.

X and ϕ gets vevs due to their potentials $V_{\phi, X}$, they fix the vev profile in the fifth dimension and the BCs that these fields obey. The condition for this potential to be natural is ¹³

$$V_X, V_\phi \sim \frac{1}{l_0^4}, \tag{A.2}$$

or, in other words, this potential generates vevs for X and ϕ with BCs,

$$X(l_0), \phi(l_0) \sim 1/l_0. \tag{A.3}$$

One cannot solve analytically the whole system. On the other hand, we are interested in the net effect on the $SU(2)_L \times SU(2)_R$ fields. Therefore, one can solve for the system gravity+ ϕ and add the effect of the charged field X .

A.1 Neutral scalar

In this section we illustrate how the coupling of a scalar to gravity can generate a metric that deviates from AdS as in eq. (3.3). The scalar does not break electroweak symmetry: its effect will be common to axial and vector resonances.

We use the ansatz [58, 59]

$$\phi = \phi(z), \tag{A.4}$$

to write down the equations of motion

$$\kappa^2 \phi'^2 = 6(A - B) \tag{A.5}$$

$$\kappa^2 V(\phi) = -\frac{3}{w^2} (A + B), \tag{A.6}$$

where we have defined

$$A(z) = 2 \frac{w'^2}{w^2}$$

$$B(z) = \frac{w''}{w}.$$

In AdS, $A = B = 2/z^2$. In a metric of the form (3.3),

$$A = \frac{2}{z^2} \left(1 - \frac{4o}{d-1} \left(\frac{z}{l_1} \right)^{2d} \right)$$

$$B = \frac{2}{z^2} \left(1 + \frac{2d-3}{d-1} o \left(\frac{z}{l_1} \right)^{2d} \right).$$

¹³See [42] for an example of V_X .

The solution is

$$\phi(z) = \phi(l_0) + \frac{2}{d} \sqrt{\frac{-3(2d+1)o}{d-1}} \left(\frac{z}{l_1}\right)^d \tag{A.7}$$

$$V(\phi) = -\frac{12}{l_0^2} e^{\frac{d}{6(2d+1)}(\phi-\phi_0)^2} \left(1 - \frac{d^2}{24} \frac{2d-7}{2d+1} (\phi-\phi_0)^2\right) \tag{A.8}$$

The first thing we notice is that

$$\phi \text{ non-tachyonic} \implies o < 0 \tag{A.9}$$

One can also check what will happen to gravity in this case. The graviton equation of motion will receive a correction from the condensates that again takes over the pure AdS near the IR:

$$-\psi'' + \frac{1}{z^2} \left(\frac{15}{4} + \delta V(z)\right) \psi(z) = m^2 \psi(z), \tag{A.10}$$

where the extra piece in the potential is given by

$$\delta V(z) = 3d\nu \left(\frac{z}{l_1}\right)^{2d} \left(4 - 2d + 3d\nu \left(\frac{z}{l_1}\right)^{2d}\right). \tag{A.11}$$

The zero mode is simply

$$\psi_0(z) = \frac{w(z)^{3/2}}{N_0}, \tag{A.12}$$

where N_0 is the norm of the graviton.

The only corrections to gravity will come from the tower of KK gravitons,

$$G_N \sim M_{5D}^{-3} \psi_0(l_0)^2. \tag{A.13}$$

A.2 Symmetry-breaking by a bulk scalar

The LR symmetry has to be broken near the IR brane. The standard way to model this would be to introduce a bulk scalar that describes the lowest dimension condensate associated with that breaking. Breaking by BCs on the other hand, would only introduce non-local order parameters. Here we show that, for our purposes, breaking by a bulk scalar is equivalent to introducing an effective metric for the axial channel, and modifying the BCs.

If a bulk scalar X transforming as a bifundamental under $SU(N_f)_L \times SU(N_f)_R$ acquires a profile $v(z)$, the wave equation for the axial KKs is modified as follows

$$-\frac{1}{w} \partial (w \partial \Phi) + 2w^2 v^2 \Phi = M^2 \Phi, \tag{A.14}$$

where the new term is the one proportional to v^2 [6, 7]. The above equation can be recast in the Schrödinger form

$$-\partial^2 \psi + V \psi = M^2 \psi, \tag{A.15}$$

provided we define the new wave-functions ψ as follows

$$\psi = \sqrt{w}\Phi. \tag{A.16}$$

We can then read off the potential $V(z)$ in the Schrödinger equation (A.15) as

$$V = \frac{\partial^2 \sqrt{w}}{\sqrt{w}} + w^2 v^2. \tag{A.17}$$

The point is now to invert the above trick of going from Φ to ψ , but for a potential given by V rather than simply by $\frac{\partial^2 \sqrt{w}}{\sqrt{w}}$. In other words, we want to solve for w_X in the second-order differential equation

$$\frac{\partial^2 \sqrt{w_X}}{\sqrt{w_X}} = V. \tag{A.18}$$

The right solution can be picked by asking that the effective warp factor w_X be asymptotically AdS, i.e. the condition

$$\frac{w_X}{w} \Big|_{z=l_0} \xrightarrow{l_0 \rightarrow 0} 1, \tag{A.19}$$

excludes the divergent linear combination, and also fixes the normalization.

The basic relation we need is then

$$\sqrt{w}\Phi = \sqrt{w_X}\varphi, \tag{A.20}$$

so that the normalization condition is now

$$\int_{l_0}^{l_1} dz w_X \varphi^2 = \mathcal{N}, \tag{A.21}$$

and (+) BCs for the original Φ wave-functions are modified into mixed ones for the φ 's

$$-\partial \log \varphi = \frac{1}{2} \partial \log \frac{w_X}{w}. \tag{A.22}$$

In the limit of a large condensate, w_X deviates strongly from w in the IR, and this tends to a (-) BC.

In an AdS background $w = l_0/z$, the differential equation (A.18) reduces to

$$\frac{\partial^2 \sqrt{w_X}}{\sqrt{w_X}} = \frac{3}{4} \frac{1}{z^2} + \left(\frac{l_0}{z}\right)^2 v(z)^2, \tag{A.23}$$

which can be solved analytically if $v(z)$ is a power-law.

$$v(z) = \sigma z^d \tag{A.24}$$

$$\frac{w_X}{w} = {}_0F_1 \left(; \frac{d-1}{d}; \frac{\sigma^2 l_0^2 z^{2d}}{2d^2} \right) \underset{z \rightarrow 0}{\sim} 1 + \frac{\sigma^2 l_0^2}{d(d-1)} z^{2d} + \mathcal{O} \left(\frac{1}{d^4} \sigma^4 z^{4d} \right) \tag{A.25}$$

For the particular case of $d = 2$

$$d = 2 \quad w_X = \frac{l_0}{z} \text{Cosh}^2 \left(\frac{\sigma z^2}{2} \right) \underset{z \rightarrow 0}{\sim} \frac{l_0}{z} \left(1 + \frac{\sigma^2}{4} z^4 + \frac{\sigma^4}{48} z^8 \dots \right) \quad (\text{A.26})$$

Adding several fields of scaling dimensions $2, 3, 4 \dots d$ would have an effect on the metric suppressed by $(z/l_1)^{2d}$. The lower the dimension, the more the deviation from AdS extends inside the bulk. The effect is of course maximum on the IR brane, but still there there is a suppression given by the dimension that is suppressed by the dimension of the field,

$$\left(\frac{1}{4}, \frac{1}{12}, \frac{1}{25}, \dots, \frac{1}{2d(d-1)} \right)$$

In conclusion, for phenomenological purposes, one only needs to consider the effect of the lower dimension condensates.

B. Derivation of S in any holographic model

We derived a sum rule for S in [60]. It applies to 5D models with BCs, where the symmetry breaking was limited to a crossed LR term, yielding different effective metrics for V and A . Here, we want to generalize this result to the case where deviations from AdS (and in particular symmetry breaking) are introduced by bulk scalars. For a quadratic quantity such as the S parameter, this can be done by using the results of appendix A.2. Indeed, there we showed how to rewrite the effect of bulk scalars as effective metrics and effective BCs.

For any wave-equation with $(-)$ UV BC, we can write the decay constants of the KK with wave-function φ_n and mass M_n as

$$\frac{g_5^2}{\sqrt{2}} f_n M_n^2 = w \partial \varphi_n |_{l_0}. \quad (\text{B.1})$$

This is true whatever the basis, i.e. using Φ , ψ or φ wave-functions. Indeed, whatever the representation used, the only non-vanishing quadratic terms remaining after using the EOMs are surface terms. Variation of these with respect to the source on the UV brane yields their coupling to the KKs, i.e. the resonance decay constants.

To be general, we then consider mixed IR BCs in the form

$$-\partial \log \varphi |_{l_1} = g_5^2 w (l_1)^2 M_{\text{IR}}^2. \quad (\text{B.2})$$

Eq. (B.1) can be recast as

$$\frac{g_5^2}{\sqrt{2}} f_n M_n^2 = w \alpha^2 \partial \left(\frac{1}{\alpha} \varphi_n \right) \Big|_{l_0}, \quad (\text{B.3})$$

provided we normalize the function $\alpha(z)$ such that

$$\alpha(l_0) = 1. \quad (\text{B.4})$$

Other than that, the function α is undetermined at that stage. The point is that, if α satisfies the same IR BC as φ (B.2), i.e.

$$-\partial \log \alpha|_{l_1} = g_5^2 w(l_1)^2 M_{\text{IR}}^2, \quad (\text{B.5})$$

and a massless spin-1 EOM

$$\partial(w\partial\alpha) = 0, \quad (\text{B.6})$$

then we can turn (B.3) into a useful expression, namely

$$f_n = \frac{\sqrt{2}}{g_5^2} \int_{l_0}^{l_1} dz w \alpha \varphi_n. \quad (\text{B.7})$$

To make use of this, we still have to give the explicit expression for α

$$\alpha(z) = \frac{\left(g_5^2 w(l_1)^3 M_{\text{IR}}^2\right)^{-1} + \int_z^{l_1} \frac{dz'}{w(z')}}{\left(g_5^2 w(l_1)^3 M_{\text{IR}}^2\right)^{-1} + \int_{l_0}^{l_1} \frac{dz'}{w(z')}}, \quad (\text{B.8})$$

which satisfies the EOM (B.6) and the two BCs (B.4) and (B.5). It turns out that α can be interpreted in the completely general case, as the wave-function of the GBs. This implies in particular that the GB decay constant is given by

$$f^2 = \frac{1}{g_5^2} w \partial \alpha \Big|_{l_0}. \quad (\text{B.9})$$

For the model with SB implemented by BCs and two different metrics for A and V , we can check that the solutions for α were respectively

$$\alpha_V \equiv 1, \quad (\text{B.10})$$

$$\alpha_A = \frac{\int_z^{l_1} \frac{dz'}{w_A(z')}}{\int_{l_0}^{l_1} \frac{dz'}{w_A(z')}}, \quad (\text{B.11})$$

since the IR BCs for V/A correspond to vanishing/infinite M_{IR} respectively.

In the case where symmetry breaking is implemented by a bulk scalar, we can rewrite the effect of the scalar vev v on the axial wave-functions Φ_A as an effective metric w_A felt by the wave-functions φ_A . We can then simply apply the method of section A.2, using w_A as the metric, and taking in to account the change of IR BC as follows.

For the transformed wave-functions φ_A , we obtain mixed BCs on the IR brane, as indicated by (A.22). Indeed, (A.22) translates into

$$M_{\text{IR}}^2 = \frac{1}{2g_5^2 w_A^2} \partial \log \frac{w_A}{w_V} \Big|_{l_1}, \quad (\text{B.12})$$

and we can then plug this into the solution for α (B.8). Applying the completeness relation for the φ_A 's with the metric w_A , we can derive

$$S = \frac{16\pi}{g_5^2} \int_{l_0}^{l_1} dz (w_V(z) \alpha_V(z)^2 - w_A(z) \alpha_A(z)^2). \quad (\text{B.13})$$

This is the general result for S , valid using the expressions for α given in (B.8) for the case of mixed IR BCs. For the case with bulk scalars, one needs to have determined beforehand the effective metric and IR mass using the techniques of section A.2.

C. NDA for the condensates

We detail here the NDA estimates for the deviations from AdS. Whereas in [29] we estimated the natural size for condensates from the 4D OPE, we propose here to start from the side of the 5D modeling. Unsurprisingly, the results essentially agree, provided we account for missing factors in [29], which do not matter for the low-dimension condensates we are interested in.

Imagine the situation of section A.2, i.e. a symmetry-breaking VEV for the scalar field X . To perform dimensional analysis, we consider an AdS background. Then, in order for this bulk profile to generate a dimension $2d$ condensate in the axial two-point function, we have to assume that its 5D mass is given by $m^2 l_0^2 = d(d-4)$. The scalar may then develop a profile of the shape $v(z) = \frac{\sqrt{c}}{l_0} \left(\frac{z}{l_1}\right)^d$ where the IR value is set by a potential localized on the IR brane. NDA on this potential (A.2) implies

$$o = \mathcal{O}(1). \tag{C.1}$$

This translates into a dimension $2d$ condensate appearing in the two-point function as

$$\Pi_A(Q^2) = -\frac{N}{12\pi^2} \log\left(\frac{Q^2}{\mu^2}\right) + \frac{\langle \mathcal{O}_{2d} \rangle}{Q^{2d}} + \dots \tag{C.2}$$

where [7]

$$\langle \mathcal{O}_{2d} \rangle = \frac{1}{\sqrt{\pi}} \frac{d}{d-1} \frac{\Gamma(d)^3}{\Gamma(d+1/2)} \frac{N}{12\pi^2} o l_1^{-2d}. \tag{C.3}$$

Compared to the 4D estimate used in [29], this provides more precise numerical factors. This includes a factorial growth with d for $d \gg 1$. It is interesting to note that the NDA analysis on the simple 5D model produces the factorial growth expected in 4D [61]. Note that the factorial growth is expected for $d \gg 1$, but not necessarily for $d \lesssim 3$: this is why the investigation in [29] did not include it, in order not to artificially enhance the effect on S . Even when this growth is included, we see from figure 5 that a significant effect on S can be achieved only with a low- d condensate.

What the factorial behavior is really telling us is that the OPE cannot be resummed. Also, one may wonder whether adding higher and higher orders by including additional scalars with a bulk profile is a convergent procedure. We can answer this question by recasting the various profiles as a deformation of the metric: this allows us to compare the respective deviations with the AdS background. It turns out that the deviation from AdS is largest on the IR brane, and goes down with d for a profile z^d/l_1^{d+1} as

$$\frac{1}{d(d-1)}. \tag{C.4}$$

Such a series can be summed, implying that the deformations from AdS, as estimated from the model with scalars can be resummed, while still leading to the (divergent) factorial growth in the OPE.

D. Comparison 4D resonance saturation models

In general, any Green's function becomes meromorphic in the large- N limit [62]. Thus, we can hope to model the two-point function by a sum of poles, located at the masses of the resonances, and with residues related to their decay constant. To get started, one may imagine a situation where the vector and axial two-point spectral functions are equal above some scale, so that $\text{Im } \Pi_{LR}$ vanishes above that scale. We then only need to consider the finite number of resonances that are below that scale. In that case, we have a finite number of parameters (the decay constants and masses of the resonances). The result for the easiest cases are as follows

- Only one resonance: the first WSR fixes it to be a vector, which implies $S > 0$.
- Two resonances: the first WSR fixes one of them to be a vector. Getting $S \leq 0$ requires the lightest resonance to be axial. Using the second WSR, one would then find $\langle \mathcal{O}_4 \rangle_{V-A} > 0$, in conflict with Witten's positivity constraint.
- Three resonances: assuming $\langle \mathcal{O}_4 \rangle_{V-A} = 0$ as in QCD, the authors of [63] have shown that it is possible to get $S \leq 0$ without encountering any of the above-mentioned problems (i.e. they have $\langle \mathcal{O}_6 \rangle_{V-A} < 0$). The spectrum they find is then: AVA .

In such modeling, there is in fact a conflict between obtaining $S \leq 0$ and satisfying Witten's positivity constraint for any even number of resonances. This is really a problem of the model itself, since in reality there should be an infinite number of resonances for large- N .¹⁴

However, the answer in such 4D models only depends on the $V - A$ condensate, not on the two condensates separately, whereas we've seen in the 5D model that the answer depended on both (see figure 6). Whereas a 5D model relates f with the axial condensates, as should be, in 4D models, it is an input parameter. In the 5D case, the cancellation of the pion pole against resonance contributions to yield a vanishing dimension-2 axial condensate is built in. The dimension-2 condensate thus automatically vanishes unless it is explicitly included in the model. This is not automatic in the 4D resonance saturation approach.

In addition to these concerns, we point out that the 5D model predicts the resonance couplings to the fermion currents, depending on the localization of the latter. In a generic 4D model, these couplings would be arbitrary.

To summarize, 4D models of resonances work in the following way: starting with the input of o_{V-A} , $\frac{M_\rho}{f}$ and $\frac{M_{a_1}}{f}$ one uses WSRs to compute S, f_{a_1} and f_ρ . Schematically,

$$\text{4D model : } \mathcal{O}_{V-A}, \frac{M_\rho}{f}, \frac{M_{a_1}}{f} \xrightarrow{\text{WSRs}} S, f_\rho, f_{a_1}$$

On the other hand, the 5D model works differently,

$$\text{5D model : } N, o_V, o_A \implies \frac{M_\rho}{f}, \frac{M_{a_1}}{f}, S, f_\rho, f_{a_1}$$

There are, of course, other advantages in using a 5D model besides parameter counting: the correspondence between 4D quantities and 5D objects is very intuitive (see section 1).

¹⁴Alternatively, for finite N , the resonances should get a finite width and the spectral function modified accordingly.

E. Link with TC scales

We try to relate the present results with previous ideas about the behavior of strong 4D theories. We explain this on the example of walking technicolor, in which case the high-energy and intermediate-energy scalings of the techni-quark condensate are different. To be specific, the OPE of $\Pi_{V,A}$ should include a dimension-6 condensate for $Q^2 \rightarrow +\infty$

$$\Pi_X(Q^2) \stackrel{Q^2 > \Lambda_*}{=} -\frac{N}{12\pi^2} \log\left(\frac{Q^2}{\mu^2}\right) + \frac{\langle \mathcal{O}_6 \rangle}{Q^6} + \dots \tag{E.1}$$

At energies below the critical scale Λ_* where the coupling constant walks, a large anomalous dimension may be generated for the techni-quark condensate [64–66] for the extreme walking case, yielding

$$\Pi_X(Q^2) \stackrel{\Lambda_* > Q^2 > \Lambda_{\text{TC}}}{\approx} -\frac{N}{12\pi^2} \log\left(\frac{Q^2}{\mu^2}\right) + \frac{\langle \mathcal{O}_4 \rangle}{Q^4} + \dots \tag{E.2}$$

for scales much larger than the confinement scale Λ_{TC} , but smaller than Λ_* . The behavior (E.2) is the one that has to be reproduced by the model, since it is the one which influences the value of S . Translating into 5D requires the identification of the confinement scale

$$\Lambda_{\text{TC}} \sim 1/l_1. \tag{E.3}$$

For the high scales, the exact translation will be model-dependent. One expects the localization of the fermion to correspond to the inverse of the scale at which they get their masses (assuming that it comes from an order one 5D coupling). Sticking to the simplest model with fermions on the UV brane, that scale is of order $1/l_0$. This would correspond to the extended technicolor scale Λ_{ETC} if one was thinking of modeling such a 4D set-up, in which case there should be more than just two flavors of techni-quarks, and one must discuss the issue of explicit breaking to lift the physical pseudo-GBs above the experimental limits [13]. We do not consider such a scenario here, but it is still useful to keep in mind the correspondence

$$\Lambda_{\text{ETC}} \sim 1/l_0. \tag{E.4}$$

One comment is in order about the respective sizes of Λ_{ETC} and Λ_* . It looks as if we made the hidden assumption $\Lambda_* > \Lambda_{\text{ETC}}$, since we have used the extreme walking approximation (E.2) up to the scale Λ_{ETC} . However, since the S parameter is a UV-independent quantity, the ordering of the two scales Λ_{TC} and Λ_* is irrelevant to the present discussion: having the switch-over from the $1/Q^6$ (z^6) behavior to the $1/Q^4$ (z^4) behavior above or below Λ_{ETC} (before or after $1/l_0$) is numerically unimportant. In figure 10, only the contributions from $S(z)$ near the UV brane would be affected, and they would remain small.

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